A GENERAL-PURPOSE HARMONIC-BALANCE APPROACH TO THE COMPUTATION OF NEAR-CARRIER NOISE IN FREE-RUNNING MICROWAVE OSCILLATORS

Vittorio RIZZOLI (1), Franco MASTRI (2), and Diego MASOTTI (2)

(1) Dipartimento di Elettronica, Informatica e Sistemistica, University of Bologna
Villa Griffone, 40044 Pontecchio Marconi, Bologna, ITALY

(2) Istituto di Elettrotecnica, University of Bologna
Viale Risorgimento 2, 40136 Bologna, ITALY

ABSTRACT

The paper discusses a novel harmonic-balance approach to the computation of near-carrier phase and amplitude noise in free-running microwave oscillators. The well-known qualitative picture of phase noise generation is quantitatively re-stated in a way compatible with the needs of general-purpose CAD. The capabilities of previously available approaches to the same problem are extended under several aspects.

INTRODUCTION

A noise analysis capability represents one of the indispensable features of a general-purpose simulator for nonlinear microwave integrated circuits. In particular, nonlinear noise analysis by the harmonic-balance (HB) technique is very effective, because frequency-domain analysis is naturally well-suited for describing the physical mechanisms of noise generation in nonlinear circuits [1]. In the last few years this topic has attained a considerable degree of maturity thanks to the efforts of several research teams throughout the world (e.g., [2], [3]), and some of the first commercial applications have began to appear. In spite of this, a general and rigorous treatment of the application of HB methodology to the noise analysis of *autonomous* nonlinear circuits, has not yet appeared in the open literature. This paper represents an attempt to fulfill this gap.

For an autonomous circuit, such as a free-running oscillator or a VCO, a noise analysis is considerably more complex than for a conventional forced circuit, and in particular, the usual noise theory based on frequency-conversion analysis [1] is not sufficient to solve the problem. This is primarily due to the fact that the physical effects of random fluctuations taking place in the circuit are qualitatively different depending on their spectral allocation with respect to the carrier. Noise components at low frequency deviations primarily produce a *frequency* modulation of the carrier, with a mean-square frequency fluctuation proportional to the available noise power. Noise components at high frequency deviations primarily produce a *phase* modulation of the carrier, with a mean-square phase fluctuation proportional to the available noise power. This qualitative picture of oscillator noise is obviously very well known (e.g., [4]). The main purpose of the paper is to show that the same picture can also be quantitatively derived from the HB equations of the autonomous circuit, and can thus be developed into a computational algorithm fully compatible with the requirements of a general-purpose CAD environment.

THE NOISE ANALYSIS ALGORITHM

Let us consider a nonlinear circuit designed to support an autonomous stable time-periodic regime of fundamental angular frequency ω_0 (*carrier*). For the purpose of HB analysis, the circuit is subdivided into a linear and a nonlinear subnetwork in the way shown in fig. 1. At first let us assume that the circuit is forced by DC sources and by a set of sinusoidal sources located at the carrier harmonics $k\omega_0$ (henceforward synthetically identified by the subscript "H"), and at the *sidebands* $\omega + k\omega_0$ (identified by the subscript "B"), where $-n_H \leq k \leq n_H$ and n_H is the number of carrier harmonics to be taken into account in the analysis. The angular frequency ω ($0 < \omega < \omega_0$) will be called the *frequency deviation* from the carrier. The electrical regime is then quasi-periodic and can be found by solving a nonlinear algebraic system of the form [5]

$$\mathbf{E}(\mathbf{X}_B, \mathbf{X}_H) = \mathbb{F} \tag{1}$$

where $\mathbf{E}$ is the vector of real and imaginary parts of all HB errors [5]. $\mathbb{F}$ is a forcing term comprehensive of DC, harmonic, and sideband excitations. In (1) the problem unknowns, consisting of the real and imaginary parts of the state-variable (SV) harmonics, have been organized in two vectors $\mathbf{X}_B$ (containing the sideband components) and $\mathbf{X}_H$ (containing components at the carrier harmonics). Similarly, the error vector $\mathbf{E}$ can be subdivided into two subvectors $\mathbf{E}_B$, $\mathbf{E}_H$.

For the analysis of the autonomous (noiseless) steady state, the forcing term $\mathbb{F}$ in (1) only contains DC sources. By assumption this solution has the form $\mathbf{X}_B = \mathbf{0}$, $\mathbf{X}_H = \mathbf{X}_H^{ss}$, where the label "ss" denotes steady-state quantities. Since the system is autonomous, the phase of the steady state is arbitrary, and the carrier frequency ω_0 represents one of the problem unknowns [5]. Thus in the vector $\mathbf{X}_H$ the phase (or the imaginary part) of one of the harmonics is replaced by ω_0.

Let us now assume that the steady state is perturbed by noise generated inside the circuit. According to [1], this situation can be described by introducing a noise voltage and a noise current source at every interconnecting port, as shown in fig. 1. These sources are comprehensive of all noise contributions generated by the linear and the nonlinear subnetwork. If the noise perturbations are small, the noise-induced deviation $[\delta\mathbf{X}_B, \delta\mathbf{X}_H]$ of the system state from the steady state $[\mathbf{0}, \mathbf{X}_H^{ss}]$ may be quantitatively described by perturbing (1) in the neighborhood of the steady state, i.e.,

$$\begin{aligned} \mathbf{M}_{BB}\,\delta\mathbf{X}_B + \mathbf{M}_{BH}\,\delta\mathbf{X}_H &= \mathbb{J}_B(\omega) \\ \mathbf{M}_{HB}\,\delta\mathbf{X}_B + \mathbf{M}_{HH}\,\delta\mathbf{X}_H &= \mathbb{J}_H(\omega) \end{aligned} \tag{2}$$

where $\mathbf{M}_{YZ} \triangleq \left(\partial\mathbf{E}_Y/\partial\mathbf{X}_Z\right)\big|_{ss}$ and Y, Z stand for B, H in any combination. Exact algorithms for the computation of the Jacobian matrices $\mathbf{M}_{YZ}$ are discussed in [5]. Since in steady-

OF1

state conditions $\mathbf{X_B} = \mathbf{0}$, by inspection of the equations reported in [5] we get $\mathbf{M_{BH}} = \mathbf{M_{HB}} = \mathbf{0}$, so that (2) reduce to

$$\mathbf{M_{BB}}\, \delta\mathbf{X_B} = \mathbb{J}_\mathbf{B}(\omega) \quad (3)$$

$$\mathbf{M_{HH}}\, \delta\mathbf{X_H} = \mathbb{J}_\mathbf{H}(\omega) \quad (4)$$

The sideband and harmonic forcing terms $\mathbb{J}_\mathbf{B}(\omega)$, $\mathbb{J}_\mathbf{H}(\omega)$ in (2) - (4) are exclusively related to the noise sources, and deserve some comments. For a spot noise analysis at an angular frequency deviation ω, the noise sources may be physically interpreted in either of two ways. They can be viewed as pseudo-sinusoids having random amplitudes and phases, and deterministic frequencies corresponding to the noise sidebands. Under this viewpoint all the forcing terms in (2) are contained in $\mathbb{J}_\mathbf{B}(\omega)$, and $\mathbb{J}_\mathbf{H}(\omega) = \mathbf{0}$. Alternatively, the sources can be viewed as sinusoids located at the carrier harmonics, which are randomly phase- and amplitude-modulated by pseudo-sinusoidal noise at frequency ω. Under this viewpoint all the forcing terms in (2) are contained in $\mathbb{J}_\mathbf{H}(\omega)$, and $\mathbb{J}_\mathbf{B}(\omega) = \mathbf{0}$. The second viewpoint, which is quasi-stationary, can intuitively be expected to hold at low frequency deviations. This will be confirmed by the discussion to follow.

Accordingly, the uncoupled equations (3), (4) provide two alternative descriptions of noise in the autonomous system [4]. Eqn. (3) describes noise as being generated by the exchange of power among the sidebands of the *unperturbed* large-signal steady state through frequency conversion taking place in the nonlinear devices. We shall refer to this mechanism as *conversion noise*. This is exactly the same situation encountered in forced circuits, and has been dealt with in depth in the technical literature (e.g., [1] - [3]). General expressions for the PM and AM noise generated by (3) are already available [1], so that this aspect will not be further considered here. Note that the phase and amplitude modulation of the carrier produced by (3) are due to the superposition on the carrier itself of a lower and an upper sideband at the same frequency deviation. This results in a mean-square phase fluctuation proportional to the available noise power [1]. Eqn. (4) describes the noise-induced jitter of the oscillatory steady state, mathematically represented in the state space by the vector $\delta\mathbf{X_H}$. Since one of the entries of $\delta\mathbf{X_H}$ is $\delta\omega_0$, this results in a frequency jitter with a mean-square value proportional to the available noise power. We shall refer to this mechanism as *modulation noise*. The associated mean-square phase fluctuation is proportional to the available noise power divided by ω^2.

For a quantitative development of (4), the forcing term $\mathbb{J}_\mathbf{H}(\omega)$ must now be specified. In a conventional (deterministic) HB analysis, $\mathbb{J}_\mathbf{H}(\omega)$ would contain the real and imaginary parts of the synchronous perturbation phasors at $k\omega_0$. For a noise analysis, the noise source waveforms may be described as sinusoids of frequencies $k\omega_0$ modulated in both amplitude and phase at a rate ω. In (4) the phasors of the deterministic perturbations are thus replaced by the complex modulation laws, each generated by the superposition of an upper and a lower sideband contribution. Referring to fig. 1, let us first introduce the equivalent Norton phasors of the noise source sidebands

$$\mathbb{J}_k(\omega) = - [\mathbf{J}_k(\omega) + \mathbf{Y}(\omega + k\omega_0)\, \mathbf{U}_k(\omega)] \quad (5)$$

where $\mathbf{Y}$ is the linear subnetwork admittance matrix. In (5) $\mathbf{J}_k(\omega)$, $\mathbf{U}_k(\omega)$ are phasors of the pseudo-sinusoids representing the noise components falling in 1 Hz bands located in the neighborhood of the sidebands $\omega + k\omega_0$. In phasor notation with a rotating factor $\exp(j\omega t)$ understood, we thus obtain

$$\mathbb{J}_\mathbf{H}(\omega) = \begin{bmatrix} \mathbb{J}_0(\omega) \\ \cdots \\ \mathbb{J}_k(\omega) + \mathbb{J}_{-k}(\omega) \\ -j\mathbb{J}_k(\omega) + j\mathbb{J}_{-k}(\omega) \\ \cdots \end{bmatrix} \quad (1 \le k \le n_H) \quad (6)$$

Eqn. (4) can now be solved for $\delta\mathbf{X_H}$ and thus for $\delta\omega_0$. Let us introduce the row matrix $\mathbf{S} = [0\ 0\ \ldots\ 1\ \ldots\ 0]$, where the nonzero element corresponds to the position of the entry $\delta\omega_0$ in the vector $\delta\mathbf{X_H}$. We obtain

$$\delta\omega_0(\omega) = \mathbf{S}\, [\mathbf{M_{HH}}]^{-1}\, \mathbb{J}_\mathbf{H}(\omega) \triangleq \mathbf{T_F}\, \mathbb{J}_\mathbf{H}(\omega) \quad (7)$$

where $\delta\omega_0(\omega)$ represents the phasor of the pseudo-sinusoidal component of the fundamental-frequency fluctuations in a 1 Hz band at frequency ω. Furthermore, a straightforward perturbative analysis of the linear subnetwork allows the perturbations on the current through the load resistor R to be linearly related to the perturbation on the state vector, $\delta\mathbf{X_H}$, obtained from (4). For the phasor of the pseudo-sinusoidal component of the load current fluctuations in a 1 Hz band at a deviation ω from $k\omega_0$, we may thus write an equation of the form

$$\delta I_k(\omega) = \mathbf{T_{Ak}}\, \mathbb{J}_\mathbf{H}(\omega) \quad (8)$$

From (7) we obtain the expression of the k-th harmonic PM noise due to modulation

$$<|\delta\Phi_k(\omega)|^2> = \frac{k^2}{\omega^2}\, \mathbf{T_F} <\mathbb{J}_\mathbf{H}(\omega)\, \mathbb{J}_\mathbf{H}^\dagger(\omega)> \mathbf{T_F}^\dagger \quad (9)$$

where $<\bullet>$ denotes the ensemble average, and $\dagger$ the conjugate transposed. Similarly, from (8) we derive the k-th harmonic AM noise-to-carrier ratio

$$<|\delta A_k(\omega)|^2> = \frac{2}{|I_k^{ss}|^2}\, \mathbf{T_{Ak}} <\mathbb{J}_\mathbf{H}(\omega)\, \mathbb{J}_\mathbf{H}^\dagger(\omega)> \mathbf{T_{Ak}}^\dagger \quad (10)$$

where I_k^{ss} is the k-th harmonic of the steady-state current through the load. Finally, the k-th harmonic PM-AM correlation coefficient is given by [1]

$$C_k^{PMAM}(\omega) \triangleq <\delta\Phi_k(\omega)\, \delta A_k^*(\omega)> = \frac{k\sqrt{2}}{j\omega\, |I_k^{ss}|}\, \mathbf{T_F} <\mathbb{J}_\mathbf{H}(\omega)\, \mathbb{J}_\mathbf{H}^\dagger(\omega)> \mathbf{T_{Ak}}^\dagger \quad (11)$$

where * denotes the complex conjugate. Eqn. (9) - (11) can also be used to derive the near-carrier RF spectrum of the noisy oscillator, i.e., the noise power spectral densities $N_k(\omega)$ at the sidebands $\omega + k\omega_0$ [1]:

$$N_k(\omega) = \frac{1}{4} R \left\{ \frac{k^2}{\omega^2} |I_k^{ss}|^2 \mathbf{T_F} <\mathbb{J}_H(\omega)\ \mathbb{J}_H^{\dagger}(\omega)> \mathbf{T_F}^{\dagger} + \right.$$

$$+ \mathbf{T_{Ak}} <\mathbb{J}_H(\omega)\ \mathbb{J}_H^{\dagger}(\omega)> \mathbf{T_{Ak}}^{\dagger} +$$

$$\left. + 2 \frac{k}{\omega} |I_k^{ss}| \operatorname{Re}[\mathbf{T_F} <\mathbb{J}_H(\omega)\ \mathbb{J}_H^{\dagger}(\omega)> \mathbf{T_{Ak}}^{\dagger}] \right\} \qquad (12)$$

The overall picture of oscillator PM noise is now evident by comparison of (9) with the results of (3). In the presence of both thermal and 1/f noise sources, conversion noise invariably raises as ω^{-1} for $\omega \rightarrow 0$, and modulation noise as ω^{-3}. The latter is consistent with experimental observations, while the former is not. Thus very near-carrier noise is essentially a modulation phenomenon and has to be described by (4) and (9). On the other hand, for $\omega \rightarrow \infty$ modulation noise vanishes due to (9), while conversion noise tends to a finite limit corresponding to the oscillator noise floor. Again, the latter is consistent with experimental observations, while the former is not. Thus very far-from-carrier noise is essentially a conversion phenomenon and has to be described by (3). Eqn. (3) and (4) necessarily yield the same evaluation of phase noise at some crossover frequency ω_x. Thus, in order to compute PM noise at any frequency deviation, an obvious criterion is to use (4) below ω_x, and by (3) above ω_x. Note that this approach is not as artificial as it might seem at a first glance, since in many practical cases (3) and (4) yield virtually identical results in a broad neighborhood of ω_x (often more than two decades). An example is given in fig. 3. The same criterion is used for computing AM noise. It is noteworthy that in many practical cases, (3) and (4) give almost identical results for AM noise at all frequency deviations, so that the two viewpoints are then essentially equivalent. An example is again given in fig. 3.

One final point is worth some consideration. In most practical cases, the noise waveforms generated by a nonlinear device are deterministic functions of the device state, possibly through some state-dependent device parameter(s). Such waveforms are thus periodically modulated by the large-signal steady state. This implies that in general the device noise sources are not stationary in the strict sense, but only wide-sense stationary, from the statistical viewpoint [6]. Thus they still admit a spectral description, which may be developed in the way outlined below. The modulation will in general produce noise source sidebands which will appear in the forcing term $\mathbb{J}_H(\omega)$ of (4) even in the case of base-band sources. The theory of noise source modulation has been developed in detail in [1], [2], and will not be repeated here. We shall only report the results of the application of this theory to a typical flicker noise current source having the DC (unmodulated) spectral density

$$<|J(\omega)|^2> = Q \frac{|I_D|^{\beta}}{\omega^{\alpha}} \qquad (13)$$

where I_D is the bias value of some current $i_D(t)$ flowing through the device, and Q, α, β are model parameters ($\alpha \approx 1$). In the presence of the large-signal oscillatory regime, $i_D(t)$ becomes a periodic function of time, and noise sidebands appear in the source spectrum. In complex phasor notation, the correlation coefficient of the p-th and q-th sidebands is given by [1]

$$<J_p(\omega)\, J_q^*(\omega)> = Q \sum_{s=-n_H}^{n_H} \frac{H_{p-s}\, H_{s-q}}{|\omega + s\omega_0|^{\alpha}} \approx Q \frac{H_p\, H_q^*}{\omega^{\alpha}} \qquad (14)$$

where H_k is the k-th harmonic of $|i_D(t)|^{\beta/2}$. The last approximate equality on the right-hand side of (14) is justified by the fast decay of the source spectral density for increasing ω. The contribution of the flicker noise source under consideration to the correlation matrix $<\mathbb{J}_H(\omega)\ \mathbb{J}_H^{\dagger}(\omega)>$ appearing in (9) - (12) can be obtained from (5), (6) through (14) in a straightforward way.

TYPICAL RESULTS

Let us consider the microstrip DRO schematically depicted in fig. 2. The circuit was designed by numerical optimization for a steady-state oscillation at $\omega_0 = 2\pi \cdot 4.6$ GHz with an output power of +12.5 dBm. For noise analysis purposes, the shot/diffusion noise in the channel is described by the usual van der Ziel model [2], and the flicker noise by a voltage source connected in series to the gate [7]. The spectral density of this source is $3.35 \cdot 10^{-9}/\omega$ V^2/Hz. Fig. 3 shows the modulation and conversion contributions to the oscillator phase noise at the nominal design point as a function of frequency deviation from the carrier. The behavior outlined in the general discussion is clearly observed. The figure also provides a comparison with the measured PM noise. The analysis is seen to accurately predict both the near-carrier PM noise and the noise floor of the oscillator. Fig. 3 also shows the computed AM noise (measured information on AM noise was not available at the time of this writing). The AM noise curves obtained from (3) and (4) are not distinguishable. The overall CPU time required to produce all the results presented in fig. 3 is about 50 seconds on a SUN SPARCstation 2.

CONCLUSION

The paper introduces a novel general-purpose approach to the noise analysis of microwave oscillators, based on the piecewise harmonic-balance principle. The new method allows the computation of far- and near-carrier noise in arbitrary user-defined circuits, without any compromise on circuit topology, nonlinear device models, and harmonic content of the periodic steady state. Noise generation in autonomous systems is shown to be originated by two independent mechanisms: direct frequency modulation of the oscillator, dominant at low frequency deviations, and conventional frequency conversion, dominant at high frequency deviations. A unified treatment of the two aspects is found to be possible as a rigorous logical consequence of the harmonic-balance equations. In addition, the analysis takes rigorously into account the non-stationarity of the device noise sources due to the modulation of the source waveforms operated by the large-signal steady state. Previously available noise analysis capabilities are extended and improved by these new techniques in several ways. Kurokawa-like approaches to oscillator noise [8] - [9] can easily be reobtained as special cases of the general mathematical treatment presented here, by application of the theory to simplified oscillator models including the quasi-monochromatic assumption. Noise analyses based on frequency-conversion theory alone [10] are shown to be insufficient for predicting the near-carrier noise in general oscillators at very low frequency deviations, and are complemented by the modulation contribution. Critical and unnecessary numerical procedures such as the derivation of the Jacobian matrix with respect to frequency [11] are completely avoided by removing the Jacobian singularity through the use of the mixed-mode HB analysis concept [5].

REFERENCES

[1] V. Rizzoli, F. Mastri, and D. Masotti, "General-purpose noise analysis of forced nonlinear microwave circuits", *Proc. MM'92* (Brighton), Oct. 1992, pp. 293-298.

[2] V. Rizzoli, F. Mastri, and C. Cecchetti, "Computer-aided noise analysis of MESFET and HEMT mixers", *IEEE Trans. Microwave Theory Tech.*, Vol. 37, Sept. 1989, pp. 1401-1410.

[3] S. Heinen, J. Kunisch, and I. Wolff, "A unified framework for computer-aided noise analysis of linear and nonlinear microwave circuits", *IEEE Trans. Microwave Theory Tech.*, Vol. 39, Dec. 1991, pp. 2170-2175.

[4] D. B. Leeson, "A simple model of feedback oscillator noise spectrum", *Proc. IEEE*, Vol. 54, Feb. 1966, pp. 329-330.

[5] V. Rizzoli and A. Neri, "Harmonic-balance analysis of multitone autonomous nonlinear microwave circuits", *1991 IEEE MTT-S Int. Microwave Symp. Digest* (Boston), June 1991, pp. 107-110.

[6] S. Benedetto, E. Biglieri, and V. Castellani, *Digital transmission theory.* Prentice-Hall: Englewood Cliffs, 1987.

[7] H. J. Siweris and B. Schiek, "A GaAs FET oscillator noise model with a periodically driven noise source", *Proc. 16th European Microwave Conf.* (Dublin), Sept. 1986, pp. 681-686.

[8] H. Rohdin, C.-Y. Su, and C. Stolte, "A study of the relation between device low-frequency noise and oscillator phase noise for GaAs MESFETs", *1984 IEEE MTT-S Int. Microwave Symp. Digest* (S. Francisco), June 1984, pp. 267-269.

[9] A. N. Riddle and R. J. Trew, "A novel GaAs FET oscillator with low phase noise", *1985 IEEE MTT-S Int. Microwave Symp. Digest* (St. Louis), June 1985, pp. 257-260.

[10] J. M. Paillot *et al.*, "A general program for steady state, stability, and FM noise analysis of microwave oscillators", *1990 IEEE MTT-S Int. Microwave Symp. Digest* (Dallas), May 1990, pp. 1287-1290.

[11] W. Anzill, F. X. Kärtner, and P. Russer, "Simulation of the single-sideband phase noise of oscillators", *Digest of Second Int. Workshop on Integrated Nonlinear Microwave and Millimeterwave Circuits* (Duisburg), Oct. 1992, pp. 97-110.

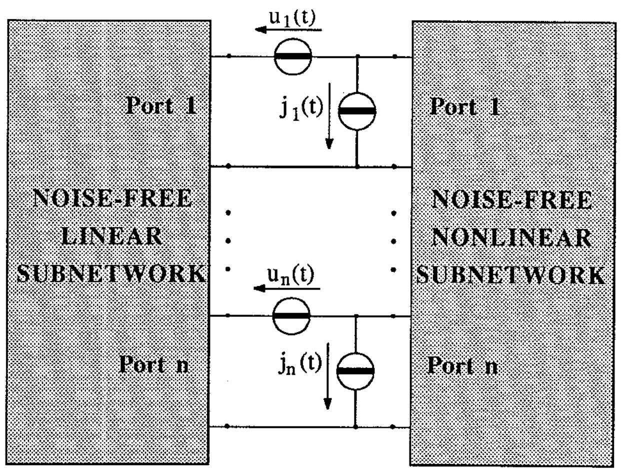

Fig. 1 - Equivalent representation of a noisy nonlinear circuit

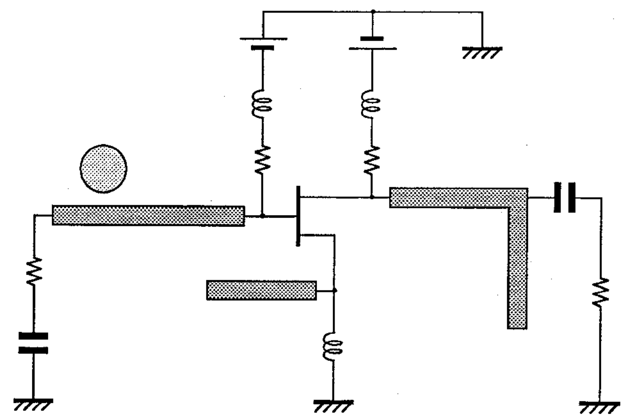

Fig. 2 - Schematic topology of a microstrip DRO

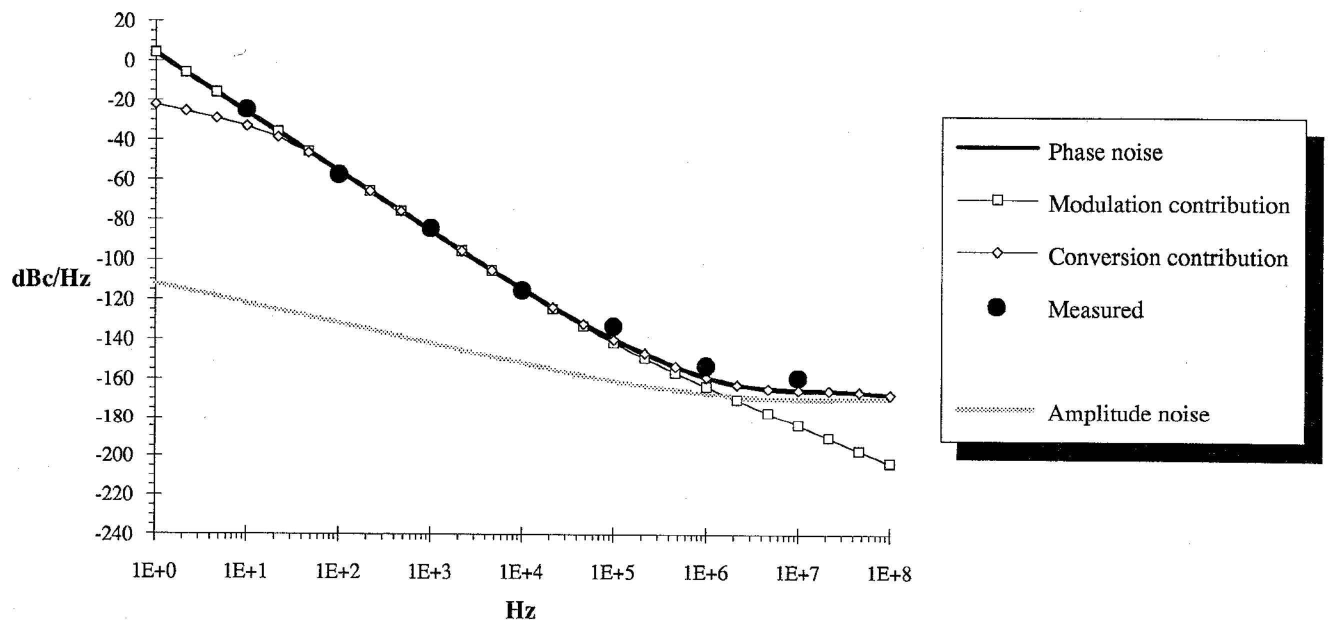

Fig. 3 - Simulated and measured near-carrier noise of a microstrip DRO